

## WEAK AND STRONG MEAN CONVERGENCE THEOREMS FOR SUPER HYBRID MAPPINGS IN HILBERT SPACES

MAYUMI HOJO\*, WATARU TAKAHASHI\*\* AND JEN-CHIH YAO\*\*\*,<sup>1</sup>

\*Graduate School of Science and Technology  
Niigata University, Niigata, Japan  
E-mail: f07n013g@mail.cc.niigata-u.ac.jp

\*\*Department of Applied Mathematics  
National Sun Yat-sen University, Kaohsiung 80424, Taiwan  
E-mail: wataru@is.titech.ac.jp

\*\*\*Department of Applied Mathematics  
National Sun Yat-sen University, Kaohsiung 80424, Taiwan  
E-mail: yaojc@math.nsysu.edu.tw

**Abstract.** In this paper, we first introduce a class of nonlinear mappings called extended hybrid in a Hilbert space containing the class of generalized hybrid mappings. The class is different from the class of super hybrid mappings which was defined by Kocourek, Takahashi and Yao [12]. We prove a fixed point theorem for generalized hybrid nonself-mapping in a Hilbert space. Next, we prove a nonlinear ergodic theorem of Baillon's type for super hybrid mappings in a Hilbert space. Finally, we deal with two strong convergence theorems of Halpern's type for these nonlinear mappings in a Hilbert space.

**Key Words and Phrases:** Hilbert space, nonexpansive mapping, nonspreading mapping, hybrid mapping, fixed point, mean convergence.

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### REFERENCES

- [1] K. Aoyama, Y. Kimura, W. Takahashi, M. Toyoda, *Approximation of common fixed points of a countable family of nonexpansive mappings in a Banach space*, *Nonlinear Anal.*, **67**(2007), 2350-2360.
- [2] J.-B. Baillon, *Un theoreme de type ergodique pour les contractions non lineaires dans un espace de Hilbert*, *C. R. Acad. Sci. Paris, Ser. A-B*, **280**(1975), 1511-1514.

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<sup>1</sup>Corresponding author.

- [3] E. Blum, W. Oettli, *From optimization and variational inequalities to equilibrium problems*, Math. Student, **63**(1994), 123-145.
- [4] F.E. Browder, *Convergence theorems for sequences of nonlinear operators in Banach spaces*, Math. Z., **100**(1967), 201-225.
- [5] P.L. Combettes, A. Hirstoaga, *Equilibrium problems in Hilbert spaces*, J. Nonlinear Convex Anal., **6**(2005), 117-136.
- [6] K. Goebel, W.A. Kirk, *Topics in Metric Fixed Point Theory*, Cambridge University Press, Cambridge, 1990.
- [7] B. Halpern, *Fixed points of nonexpanding maps*, Bull. Amer. Math. Soc., **73**(1967), 957-961.
- [8] T. Ibaraki, W. Takahashi, *Weak convergence theorem for new nonexpansive mappings in Banach spaces and its applications*, Taiwanese J. Math., **11**(2007), 929-944.
- [9] T. Ibaraki, W. Takahashi, *Fixed point theorems for nonlinear mappings of nonexpansive type in Banach spaces*, J. Nonlinear Convex Anal., **10**(2009), 21-32.
- [10] S. Iemoto, W. Takahashi, *Approximating fixed points of nonexpansive mappings and nonspreading mappings in a Hilbert space*, Nonlinear Anal., **71**(2009), 2082-2089.
- [11] S. Itoh, W. Takahashi, *The common fixed point theory of single-valued mappings and multi-valued mappings*, Pacific J. Math., **79**(1978), 493-508.
- [12] P. Kocourek, W. Takahashi, J.-C. Yao, *Fixed point theorems and weak convergence theorems for generalized hybrid mappings in Hilbert spaces*, Taiwanese J. Math., to appear.
- [13] F. Kohsaka, W. Takahashi, *Existence and approximation of fixed points of firmly nonexpansive-type mappings in Banach spaces*, SIAM J. Optim., **19**(2008), 824-835.
- [14] F. Kohsaka, W. Takahashi, *Fixed point theorems for a class of nonlinear mappings related to maximal monotone operators in Banach spaces*, Arch. Math., **91**(2008), 166-177.
- [15] Y. Kurokawa, W. Takahashi, *Weak and strong convergence theorems for nonspreading mappings in Hilbert spaces*, Nonlinear Anal., to appear.
- [16] W.R. Mann, *Mean value methods in iteration*, Proc. Amer. Math. Soc., **4**(1953), 506-510.
- [17] W. Takahashi, *A nonlinear ergodic theorem for an amenable semigroup of nonexpansive mappings in a Hilbert space*, Proc. Amer. Math. Soc., **81**(1981), 253-256.
- [18] W. Takahashi, *Nonlinear Functional Analysis*, Yokohoma Publishers, Yokohoma, 2000.
- [19] W. Takahashi, *Convex Analysis and Approximation of Fixed Points*, (Japanese), Yokohama Publishers, Yokohama, 2000.
- [20] W. Takahashi, *Introduction to Nonlinear and Convex Analysis*, Yokohoma Publishers, Yokohoma, 2009.
- [21] W. Takahashi, *Fixed point theorems for new nonlinear mappings in a Hilbert space*, J. Nonlinear Convex Anal., **11**(2010), 79-88.
- [22] W. Takahashi, *Nonlinear operators and fixed point theorems in Hilbert spaces*, RIMS Kokyuroku, **1685**(2010), to appear.
- [23] W. Takahashi, M. Toyoda, *Weak convergence theorems for nonexpansive mappings and monotone mappings*, J. Optim. Theory Appl., **118**(2003), 417-428.
- [24] W. Takahashi, J.-C. Yao, *Fixed point theorems and ergodic theorems for nonlinear mappings in Hilbert spaces*, Taiwanese J. Math., to appear.
- [25] W. Takahashi, J.-C. Yao, P. Kocourek, *Weak and strong convergence theorems for generalized hybrid nonself-mappings in Hilbert spaces*, J. Nonlinear Convex Anal., to appear.
- [26] R. Wittmann, *Approximation of fixed points of nonexpansive mappings*, Arch. Math., **58**(1992), 486-491.

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